1   INTRODUCTION

Each time a part rotates, a rolling bearing might be involved – from small dental drills to huge wind turbines. In general, a roller bearing separates two components, which can be rotated relative to one another, while still allowing a flow of force between both parts. In mechanical systems rolling bearings are often mounted to a shaft or an axel and into a housing. As rolling bearings are highly stressed machine elements, their fatigue life could determine the reliability and service life (Guo et al., 2015) of such mechanical systems and therefor the maintenance costs.

Roller bearings normally possess internal clearance as well as clearance due to a loose fit of the bearings rings. Generally, such clearance could have a negative effect on the service life of machine components. For instance, the fatigue life of a rolling bearing depends – among other things – on the operating internal clearance according to ISO/TS 16281 (2008). This correlation has also been described by Oswald, Zaretsky and Poplawski (2009, 2012) who performed various computational analyses on interference fitted low-speed cylindrical roller bearings and deep-groove ball bearings. The authors show that the fatigue life of radially loaded roller and ball bearings can significantly increase or decrease due to the internal clearance of a bearing. For instance, a small negative internal clearance (preloading) has a positive effect on the fatigue life. However, if there is too much preloading a rapid decline of the fatigue life occurs. On the other hand, an increasing positive internal clearance also decreases the fatigue life. Hence, an engineering designer must act diligently when choosing a rolling bearing and determining a bearing seats in order to achieve an operating clearance that prolongs a rolling bearings fatigue life.

Ye and Wang (2015) suggest a detailed calculation method for the optimization of the fatigue life of cylindrical roller bearings. The method is based on a quasi-dynamic model of bearings. Besides tilting of the bearing components, thermal expansion and centrifugation this method also allows the consideration of the assembly interference between shaft and inner ring. After evaluating an optimal operating clearance, the calculation method could be used to determine the optimal assembly interference between shaft and inner ring. However, the authors don't specify how dimensional tolerances of the bearing seat should be specified. The fit between the housing bore and the outer ring is disregarded too. However, the rings of cylindrical roller bearings could be separately mounted, allowing a tight fit for both rings (Eschmann, Hasbargen and Weigand, 1985). According to the recommendations of the bearing manufacturers (e. g. Schaeffler (2012) or SKF (2014)) especially for circumferential load on the outer ring, a press fit between the housing bore and the outer ring is more important than a tight fit on the inner ring.

What is more, the authors neglect geometric deviations of the bearing components as well as of the adjacent components. But geometric deviations are observable on each component due to an imprecision that is inherent to all manufacturing processes (Zhang et al., 2011). In the case of roller bearings, geometric deviations on the raceways or the rolling elements could affect the initial clearance of a roller bearing. Geometric deviations on the connecting surfaces of the bearing components and on their adjacent components may also affect the behavior of the bearing seat. These uncertainties make it hardly possible to achieve a certain value for the assembly interference and the operating clearance. Geometric dimensioning and tolerancing (GD&T) at least limits the allowable geometric deviations a priori. Nevertheless, it is often difficult to set these limits, as the functional behavior of the components could be influenced in many ways (Schleich and Wartzack, 2013).

In this article a method is presented that allows the analysis of the geometric dimensioning and tolerancing of a bearing seat for cylindrical roller bearings with respect to their fatigue life. The method could therefore assist an engineering designer when determining the tolerance specification of a bearing seat. The method adds the calculation of the reference rating life (ISO/TS 16281, 2008) and the deformation of the bearing rings to the concept for the consideration of geometrical deviations in the evaluation of bearing clearance (Aschenbrenner and Wartzack, 2016). After some basic information about cylindrical roller bearings in section 2, a description of the method is given in section 3. Afterwards the application of the method is presented for a use case in section 4. The paper closes with a conclusion and an outlook in section 5.
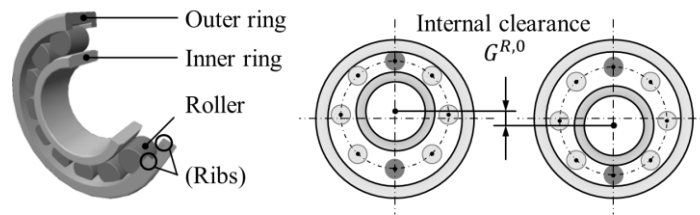
2 BASICS ABOUT CYLINDRICAL ROLLER BEARINGS

Figure 1. a) General structure of a cylindrical roller bearing
b) Illustration of the radial internal clearance of a roller bearing

Figure 1 a) shows the general structure of a cylindrical roller bearing. Although several cylindrical roller bearing types exist (e. g. N, NU, NJ or NUP), they all have in common that they consist of an outer ring, an inner ring and several rollers rotating between the two bearing rings. In order to guarantee the rolling behavior of the rollers, each bearing ring provides a very precise raceway. Ribs ensure axial guidance of the rollers. Depending on the type of cylindrical roller bearing the raceways could be surrounded by none, one or two ribs. Because of the relatively large contact area, cylindrical roller bearings can absorb high radial forces. But only those cylindrical roller bearing types having ribs on both bearing rings could deal with (comparably small) thrust loadings at all (namely NJ, NF, NH and NUP). Hence, cylindrical roller bearings are often used as floating bearings. Cylindrical roller bearings can be disassembled. Therefore the bearing rings could be separately mounted with a tight fit onto the shaft and into the housing. The dimensional tolerances for the exterior dimensions of radial rolling bearings are standardized and classified in ISO 492 (2014).

The initial bearing clearance of radial rolling bearings is standardized too (ISO 5753-1 (2009)). Actually, roller bearings can be categorized by their clearance class. As pictured in Figure 1 b), the internal bearing clearance can be defined as the distance for which the inner ring can be moved relatively to the outer ring. In general, one must distinguish between the initial clearance, the installation clearance and the operating clearance. The initial clearance is influenced by the manufacturing and assembly of the bearing components. Bearing manufactures perform elaborate steps during these processes, like specific grinding or sorting of the bearing components, to produce rolling bearings of a particular initial bearing clearance. Beside uncertainties related to the exterior dimensions of a rolling bearing the installation clearance is mainly influenced by the geometric deviations of a rolling bearing's adjacent components (namely shaft and housing). Due to the relatively low wall thickness of the bearing rings they can deform during the mounting process. For instance, an outer ring will contract and an inner ring will expand when mounted with a tight fit. Therefore the installation clearance is normally smaller than the initial clearance. What is more, the geometric deviations of the adjacent components can transfer to the raceway (Eschmann, Hasbargen and Weigand, 1985) and overlay with existing geometric deviations. In general, geometric deviations on the raceways could negatively influence the behavior of a rolling bearing in terms of vibration or load distribution (e. g. Wardle (1988), Harsha, Sandeep and Prakash (2003) etc.). The operational clearance mainly depends on the thermal gradient within a bearing. Normally the inner parts (i. e. shaft and inner ring) are warmer than the outer parts (i. e. outer ring and housing) because of an increased heat transfer of the housing (e. g. heat conduction, thermal radiation etc.). The hotter inner parts have a greater thermal expansion than the cooler outer parts, which causes further reduction of the bearing clearance. Hence, the operational clearance is usually smaller than the installation clearance.

As mentioned previously, the internal bearing clearance influences the fatigue life of a rolling bearing according to the calculation formulas of the reference rating life in ISO/TS 16281. In general, a lamina model that cuts all rollers into several laminae is used. For each lamina the dynamic load rating and dynamic load is calculated. The dynamic load rating of a lamina can be easily calculated using the calculation rules provided in ISO/TR 1281-1 (2008) and the information of the bearing manufactures regarding the dynamic load rating of a specific bearing type. For the determination of the dynamic loading of a roller lamina a force and momentum equilibrium are iteratively solved. Due to tilting and a surface profile of the bearing components (such as crowning of rollers) an increased stress can occur in those laminae which are near to an edge of a roller. For standard barrel-like crowns a formula for the increased edge stress is provided. For more complex surface profiles an advanced numerical solution for contact problems should be applied (such as: De Mul, Kalker and Frederiksson, 1986).

3 METHOD FOR THE TOLERANCE ANALYSIS OF ROLLER BEARING OPERATING CLEARANCE AND FATIGUE LIFE

According to Dantan et al. (2012) a tolerance analysis has to deal with the representation of the geometrical deviations, the calculation of the behavior of a system with deviating components and the utilization of proper analysis methods (such as worst-case searching or statistical analysis). As shown in Figure 2, the structure of the herein presented method follows this threefold division.
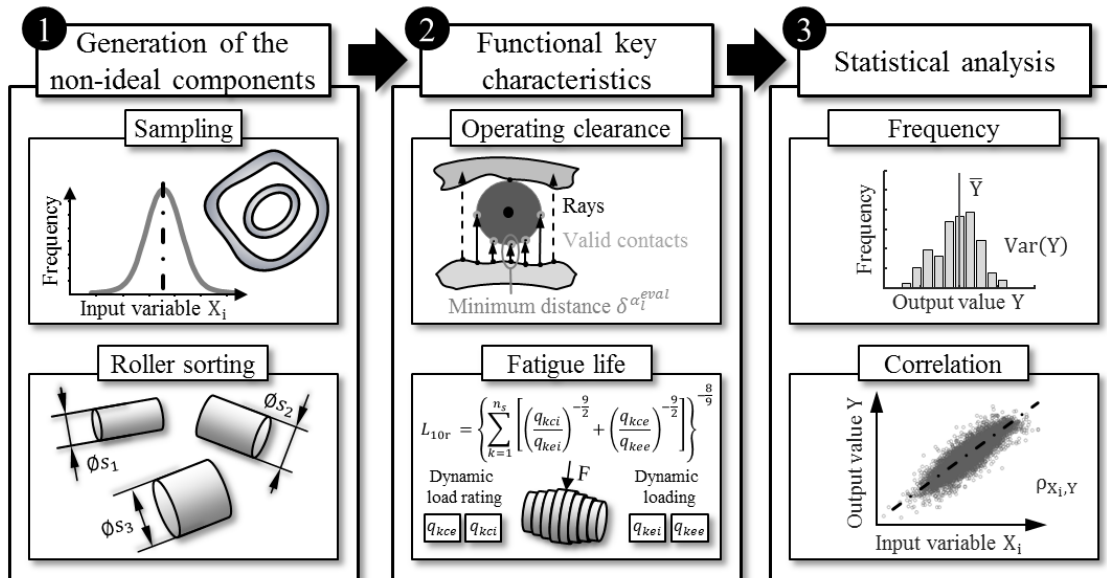

Figure 2. Necessary steps for the tolerance analysis of the operating clearance and fatigue life of cylindrical roller bearings

First of all, a large number of virtual components with their specific dimensional and geometric deviations are sampled. Utilizing the dimensions of the raceways a roller sort is selected for each roller bearing, whereby deviations may also occur within a roller sort. After the generation of the non-ideal components the operating clearances of all virtual roller bearings are calculated. These results are then used to determine the fatigue life of each roller bearing. Operating clearance and fatigue life define functional capabilities of cylindrical roller bearings and are therefore considered as functional key characteristics (FCKs) (Thornton, 1999) For both FCKs a statistical analysis is performed in which statistical characteristics are evaluated, such as the sample mean, the sample standard deviation and the correlation coefficients. Finally, this information can be used to purposefully adjust the tolerance specification of the bearing seat in order to increase the fatigue life of the roller bearings. In the following three subsections each of the first three steps is described in greater detail.

3.1 Generation of the non-ideal components

Following Schleich and Wartzack (2014), the non-ideal geometry is represented by a surface mesh. As only a 2D-radial cut of each bearing is considered, the surface mesh consists of vertices and edges. For the edges the degree of the trial function can be chosen. However, computation time could increase tremendously with the degree. Therefore only linear edges are utilized. The vertices are equidistantly distributed on the surfaces of the components (in terms of angular distance). The radial coordinate of each vertex could be obtained using mathematical descriptions such as the Discrete Fourier Transforms (Colosimo et al., 2004).

For the generation of the non-ideal components the geometric and dimensional deviations of the bearing rings, the shaft and the housing bore are sampled for each roller bearing. Bearing manufactures normally try to manufacture bearing components in such a manner, that the actual initial bearing clearance is near to the mean initial bearing clearance of a specific clearance class. For this purpose the rolling elements are classified by their diameter, whereby each class has very tight specification limits. For instance, for roller diameters smaller than 26 mm the maximum allowable radial deviation is less than 1 µm (DIN 5402-1, 2014). The actual raceway diameters can be used to

select a roller diameter class for each roller bearing individually. In practice, however, only a couple of classes are considered for reasons of capacity.

When the deviations of all components have been sampled, the mounting of the bearing rings could be simulated. In general, a deformation of the bearing rings will only occur if there are interferences between the bearing rings and their adjacent components. In this case the deformation could be evaluated using a Finite Element Simulation. However, for statistical reliable results a large number of rolling bearings must be evaluated and Finite Element Simulations come with high computational costs. At least the dimensional deformation of the bearing rings (i. e. contraction of the outer ring and expansion of the inner ring) can be easily approximated analytically following the calculation rules in DIN 7190-1 (2013). These calculation rules are based on the calculation of the cylinder stress in thin- or thick-walled cylinders (depending on the diameter ratio) (Schmid, Hamrock and Jacobson, 2014). Unfortunately, geometric deviation like out-of-roundness can't be considered. Therefore a modified calculation formula is used: Utilizing the discrete geometry representation, the components are subdivided into very thin slices. The actual interference is calculated for each slice. Next, each slice is treated like a closed cylinder and the radial deformation of this cylinder is calculated according to the calculation rules for interference fits. The results of each slice are then transferred to the respective raceway. This modification was compared to several Finite Element Simulations. As the modification tends to overestimate the deformation of the bearing rings, a linear regression model based on the results of the Finite Element Simulation was implemented. The results of the regression model are quite promising (coefficient of prognosis up to 99.70 % and a mean absolute deviation around 0.01 μm). However, the modification has only been tested for harmonic out-of-roundness deviations (waviness). Thus, further testing is necessary.

Besides the deformation of the bearing rings due to mounting, the thermal expansion of the bearing components must be considered too. For this purpose the linear thermal expansion of the components is calculated. According to Mitrović et al (2015) the temperatures of the bearing components stays almost constant after run-up. As a result, the temperature of a bearing component is the same for all evaluated roller bearings.

3.2 Determination of the functional key characteristics

For the calculation of the operating clearance the concept presented in Aschenbrenner and Wartzack (2016) is used. It follows the approach of Schleich and Wartzack (2014) which uses a ray trace algorithm for the analysis of a spur gear's run out deviations. The process for the evaluation of the operating clearance is depicted in Figure 3:
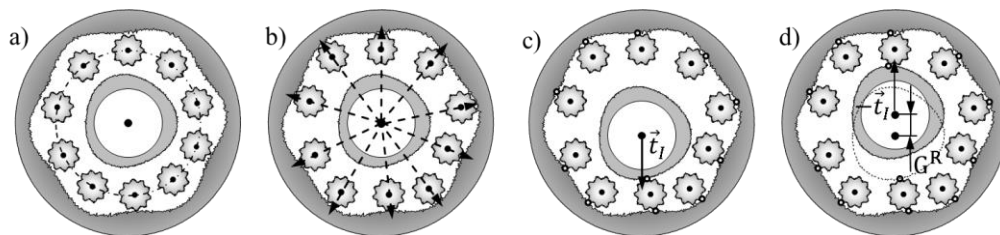
Figure 3. Process for the evaluation of the operating clearance:
a) Initial positioning of deformed non-ideal bearing components
b) Registration of the roller on the outer raceway
c) Registration of the inner ring on the roller(s) in a predefined direction
d) Registration of the inner ring on the roller(s) in the opposing direction

First of all, the non-ideal bearing components are positioned around a fixed center point with the rollers placed onto the mean bearing diameter in an angular distance equal to the separation angle. In the following steps, the bearing components are registered on each other. For the registration contact detection algorithms are employed. According to Kockara et al. (2009) those algorithms can be distinguished between broad-phase contact detection and narrow-phase contact detection. Broad-phase algorithms are used to preselect bodies or body features that might be or get in contact. For this purpose, the real bodies are substituted with simple generic volume elements, like circular or quadratic hit-boxes in 2D. The computational costs for the contact and collision detection of these substitutes are very low. In the case of the registration of the bearing components the fact is used that the polar coordinates of each surface point as well as the center points of the rollers are known or can be easily

determined ex ante in reference to the fixed central point. As shown in Figure 4, the actual geometry of a roller is substituted by an enveloping circle.
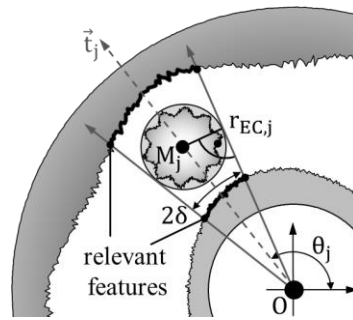

Figure 4. Broad-phase algorithm for the registration of the bearing components

The radius of the enveloping circle $r_{EC,j}$ can be used to calculate the angle δ_j within the right-angled triangle described by the point of contact of a tangential vector, the fixed center point and the center point of the roller. The relevant features (i. e. a vertex or an edge) are all those features located within the arc between $\theta_j - \delta_j$ and $\theta_j + \delta_j$. As depicted in Figure 3 c) and d) the inner ring may not have contact to all rollers at once. Thus, for a given translation direction a pre-selection of the considered rollers could be done based on the separation angle of the bearing.

Afterwards, the selected bodies respectively body features can be used in a narrow-phase algorithm to determine the actual place of contact. Herein a ray-trace algorithm is used as the narrow phase algorithm because it is suitable for surface meshes. Following Havel and Herout (2010) a ray is emitted from each surface vertex pointing into a predefined direction. If the ray hits another body a point of intersection as well as the distance between the emitting vertex and the point of intersection can be determined. The point of intersection with the smallest distance is the actual contact point. The ray-trace algorithm is employed for both, the registration of the rollers onto the outer ring and the registration of the inner ring onto the rollers. As shown in Figure 3 d), the operating clearance for a given direction is the total translation of the inner ring in this direction (which corresponds to a two-point measurement). Since the value of the operating clearance strongly depends on the selected direction, the operating clearance is evaluated for multiple directions.

These results are condensed by calculating the mean operating clearance of each roller bearing. Thereafter, the results are used to determine the fatigue life of the roller bearings. As previously described in Section 2, the reference rating life according to ISO/TS 16281 is calculated. Unfortunately, the actual profile of the bearing components hasn't been considered yet. Thus, only standard barrel-like crowns of the rollers are regarded utilizing the formula in ISO/TS 16281.

3.3 Statistical analysis of the functional key characteristics

In order to compare different settings for the geometric dimensioning and tolerancing of a bearing seat histograms as well as statistical characteristics like sample mean, sample standard deviation, sample minimum and sample maximum can be used. A way to illustrate the relationship between two variables (input and/or output) is a scatter plot. However, for a purposeful adjustment of a bearing seat's tolerance specification not only knowledge about the kind of relationship but also about the strength of the relationship between two variables is necessary. Both aspects can be described using a sensitivity analysis which characterizes how the variation of an input variable influences the variation of an output variable. In Saltelli, Chan and Scott (2000) a collection of sensitivity analysis is presented. However, the sorting of the rollers lead to statistical dependencies making it hard to implement sensitivity analysis that depend on a specific sampling like the (extended) Fourier Amplitude Sensitivity Test. Therefore the Spearman Correlation has been chosen. It is a rank correlation describing the strength of the monotonic relationship between two variables. The value of the Spearman Correlation ranges from -1 to 1, where -1 is a strong monotonous decreasing relationship and 1 is a strong monotonous increasing relationship. A value near 0 can represent a rather random correlation and therefore a weak relationship. The Spearman Correlation is used to evaluate the influence of the different deviations on the operating clearance. Nevertheless, scatter plots should also be used to identify conspicuous correlations.

4 APPLICATION OF THE METHOD FOR A SIMPLE USE CASE

For the use case a cylindrical roller bearing of the type NU206 with clearance class CN and tolerance class PN is considered. The target value of the initial bearing clearance is 32.5 µm. The roller bearing is exposed to a constant dynamic loading of 3000 N which applies as a circumferential load on the outer ring and as a point load on the inner ring. According to the bearing type the dynamical load rating is 45 kN. The temperature conditions within the bearing are considered constant (inner ring: 72.5°C, rollers: 70°C and outer ring 67.5°C). The rotational speed should be constant at 1800 rpm.

In this use case only harmonic out-of-roundness deviations (waviness) on the shafts and housing bores are considered. For comparability, all shafts should have the same number of waves ($f^S = 7$). Also the number of waves for the housing bores is constant ($f^B = 6$). The dimensional tolerances and roundness tolerances are chosen according to the recommendations of the bearing manufactures (e. g. Schaeffler (2012), SKF (2013)).

For the bearing components only dimensional deviations are considered. The exterior dimensions of the roller bearings correspond to the values in ISO 492. The interior dimensions (namely the raceway and roller diameters) are a matter for the bearing manufactures. However, the dimensional tolerances of the adjacent components of needle roller and cage assemblies can be assumed as reference values. For the rollers three diameter classes are used. The classes form a closed interval for which the values should be normally distributed across all classes. Except for the installation angles, all other values should be normally distributed centered at their mean value. Table 1 gives an overview of all ranges and statistical characteristics for all input variables.

Table 1. Variable values of the initial tolerance specification

Input variable	Range	Statistical characteristics
Shaft diameter	S: [29.980 mm; 29.993 mm]	$\mu = 29.9865$ mm; $\sigma = 2.1\overline{6}$ µm
Roundness deviation (shaft)	a^S: [0 µm; 4.5 µm]	Bisected normal distribution $\mu = 0$ µm; $\sigma = 1,5$ µm
Installation angle (shaft)	θ^S: [0°; 360°[	Uniform distribution
Housing bore diameter	B: [61.970 mm; 62.000 mm]	$\mu = 61.985$ mm; $\sigma = 5$ µm
Roundness deviation (housing)	a^B: [0 µm, 6.5 µm]	Bisected normal distribution $\mu = 0$ µm; $\sigma = 2.1\overline{6}$ µm
Installation angle (shaft)	θ^B: [0°; 360°[	Uniform distribution
Housing outer diameter	A: [94 mm; 96 mm]	$\mu = 95$ mm; $\sigma = 0.\overline{3}$ mm
Inner ring bore diameter	d: [29.990 mm; 30.000 mm]	$\mu = 29.99\overline{3}$ mm; $\sigma = 1.\overline{1}$ µm
Raceway diameter (inner ring)	F: [37.489 mm; 37.500 mm]	$\mu = 37.496\overline{3}$ mm; $\sigma = 1.\overline{2}$ µm
Raceway diameter (outer ring)	E: [55.501 mm; 55.529 mm]	$\mu = 55.510\overline{3}$ mm; $\sigma = 3.\overline{1}$ µm
Outer ring outer diameter	D: [61.987 mm; 62.000 mm]	$\mu = 61.995\overline{6}$ mm; $\sigma = 1.\overline{4}$ µm
Roller diameter	s_1: [8.988 mm; 8.990 mm[	Normal distribution across all diameter classes $\mu \approx 8.991$ mm; $\sigma = 1$ µm
	s_2: [8.990 mm; 8.992 mm[	
	s_3: [8.992 mm; 8.994 mm]	

According to Figure 5 a) the mean operating clearance is quite high for this tolerance specification of the bearing seats. Hence, a tighter fit of the bearing seats is possible and would potentially increase the fatigue life of the bearings (displayed in Figure 5 b)). This could be easily achieved by shifting the tolerance zone (and therefore the mean value) of the shaft diameter and/or the housing bore diameter.

Next, the sample standard deviation of the operating clearance is quite high leading to suboptimal constellation with an operating clearance even higher than the targeted initial clearance. Therefore the sample standard deviation should be decreased too. For this purpose one or more tolerances must be restricted. However, a restriction of the tolerances could increase manufacturing costs (Hoffenson, Dagman and Söderberg, 2013). As a result the tolerance with the highest sensitivity should be altered, because a small restriction of a highly sensitive tolerance could be more effective than a large restriction of a tolerance with a quite small sensitivity. According to Figure 5 c) the housing bore diameter has by far the highest influence on the operating clearance. Thus, the tolerance of the housing bore diameter is restricted.
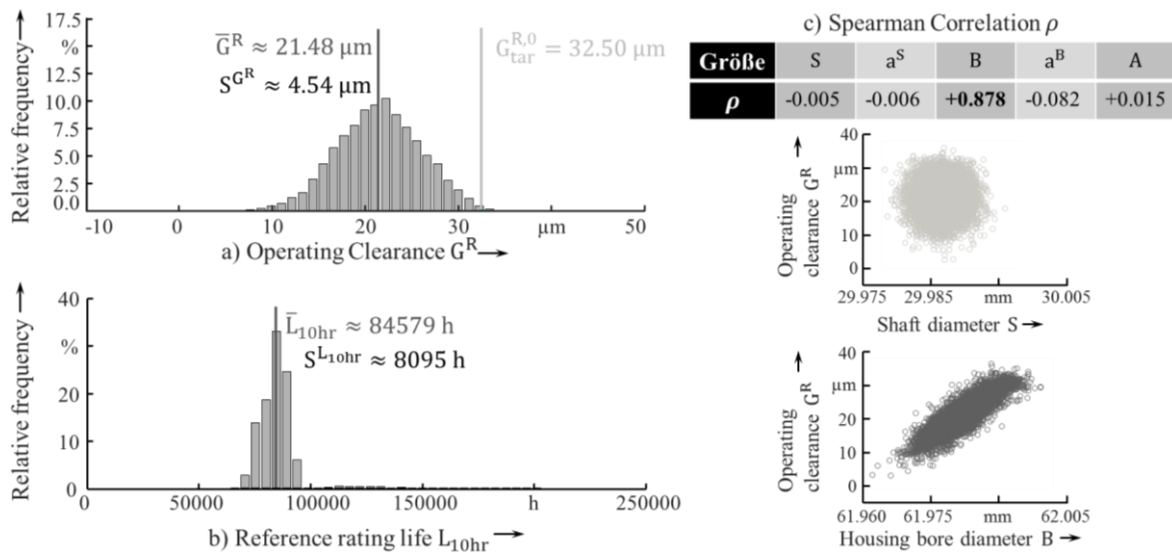
Figure 5. Results of the initial tolerance specification:
a) Histogram of the operating clearance b) Histogram of the reference rating life
c) Spearman Correlations and scatter plots of the adjacent components

Nevertheless, a closer look to the Spearman Correlations in Figure 5 c) reveals, that all the other tolerances have almost no influence on the operating clearance. The shift of the shaft diameter's tolerance zone should already increase the sensitivity of the shaft diameter. The tolerance of the outer diameter of the housing is already pretty high. Further expansion of the tolerance limits will not influence the operating clearance, but it could also have no effect on the manufacturing cost. At worst, it could even deteriorate the product quality. Therefore this value is not altered. On the other hand, the roundness tolerances are quite strict limitations with only a small influence on the operating clearance. Hence, the values of the roundness tolerances are expanded. The resulting alterations are summarized in Table 2 and the corresponding results are shown in Figure 6:

Table 2. Values of the improved tolerance specification

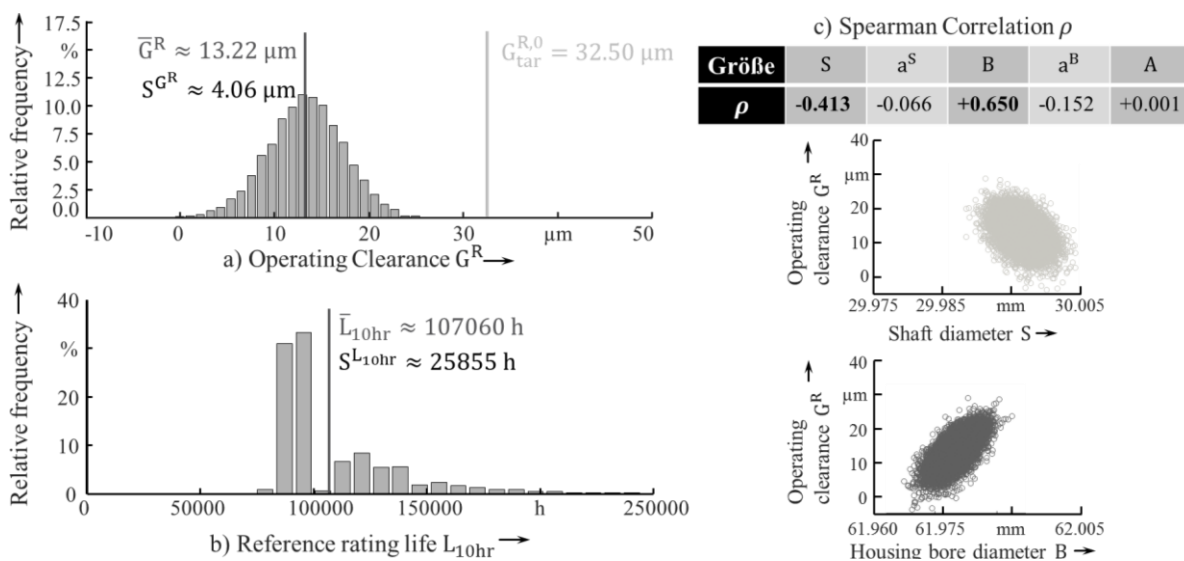| Input	Altered Range	Statistical alterations
Shaft diameter	S: [29.990 mm; 30,003 mm]	$\mu = 29.9965$ mm
Roundness deviation (shaft)	a^S: [0 µm; 6 µm]	$\sigma = 2$ µm
Housing bore diameter	B: [61.970 mm; 61,990 mm]	$\mu = 61.98$; $\sigma = 3.\bar{3}$ µm
Roundness deviation (housing)	a^B: [0 µm; 9 µm]	$\sigma = 3$ µm

Figure 6. Results of the improved tolerance specification:
a) Histogram of the operating clearance b) Histogram of the reference rating life
c) Spearman Correlations and scatter plots of the adjacent components

As expected, the interference fit on the shaft as well as the mean shift of the housing bore diameter has led to a decrease of the operating clearance. The restriction of housing bore diameter has also caused a decrease of the sample standard deviation. However, the decrease is relatively small because of the increased sensitivity of the operating clearance in reference to the other deviations (cf. Figure 5 c)). Despite of this, most of the operating clearances are still positive guaranteeing the mountability of the bearings. Moreover, the very restrictive roundness tolerances of the shaft and housing boring can be expanded.

Nevertheless, a raise of the sample standard deviation of the fatigue life occurred. A reason for this is the discontinuity of the relationship between the operating clearance and the fatigue life. Yet, the mean fatigue life has been increased tremendously.

Summing up the results for this use case, an improved tolerance specification of the bearing seats can be achieved by employing the herein presented method, though the solution is still not optimal. Further adjustments would be necessary. Instead of performing those adjustments manually an optimization algorithm could be used such as the Particle Swarm Optimization proposed by Walter, Spruegel and Wartzack (2014).

5 CONCLUSION AND OUTLOOK

In this article a method was presented which can be used for the analysis of the tolerance specification of the bearing seats for cylindrical roller bearings. The considered functional key characteristics are the operating clearance as well as the fatigue life. In the evaluation of the operating clearance not only the geometric deviations but also the deformations of the bearing components due to tight fits and thermal expansion are regarded. The operating clearances are determined using a discrete geometry representation in conjunction with contact detection algorithms. Thereafter the results of the operating clearance are coupled with the calculation of the reference rating life according to ISO/TS 16281. The method also proposes some statistical analysis in order to assist an engineering designer when altering the tolerance specification of a bearing seat. The application of the method was shown for a simple use case afterwards. For the use case an increase of the fatigue life with decreasing operating clearances could be observed. This provokes tight fits for the bearing seats.

However, an engineering designer should always consider a system at a whole. Therefore further extensions of the method are necessary like the consideration of additional key characteristics such as the vibrational behavior. What is more, not only cylindrical roller bearings but different bearings types should be considered. This implies a 3D treatment of the bearings, also allowing the consideration of a tilting due to mounting as well as a misalignment between two bearing seats on a shaft respectively in a housing.

REFERENCES

Aschenbrenner, A., Wartzack, S. (2016), "A Concept for the Consideration of Dimensional and Geometrical Deviations in the Evaluation of the Internal Clearance of Roller Bearings", *Procedia CIRP*, Vol. 43, pp. 256-261. http://dx.doi.org/10.1016/j.procir.2016.02.003

Colosimo, B. M., Moroni, G., Petro, S., Semeraro, Q. (2004), "Manufacturing Signature of Turned Circular Profiles", *Manufacturing, Modelling, Management and Control*, Vol. 15 No. 2, pp. 1–6.

Dantan, J.-Y., Gayton, N., Etienne, A., Quereshi, A. J. (2012), "Mathematical issues in mechanical tolerance analysis", 13. *Colloque National AIP PRIMECA*, Le Mont-Dore/France, March 2012, pp. 1–12. https://hal.archives-ouvertes.fr/hal-00756539

De Mul, J., Kalker, J. J., Frederiksson, B. (1986), "The Contact Between Arbitrarily Curved Bodies of Finite Dimensions", *Journal of Tribology*, Vol. 108 No. 1, pp. 140–148. http://dx.doi.org/10.1115/1.3261134

DIN 5402-1 (2014): *Rolling bearings – Pars of rolling bearings – Part 1: Cylindrical rollers*, Deutsches Institut für Normung, Berlin/Germany.

DIN 7190-1 (2013): *Interference fits – Part 1: Calculation and design rules*, Deutsches Institut für Normung, Berlin/Germany.

Eschmann, P., Hasbargen, L., Weigand K. (1985), *Ball and Roller Bearings – Theory, Design and Application*, John Wiley and Sons, Chichester, New York, Brisbane, Toronto and Singapore.

Guo, W., Cao, H., He, Z., Yang, L. (2015), "Fatigue Life Analysis of Rolling Bearings Based on Quasistatic Modeling", *Shock and Vibration*, Vol. 2015 No. 15, pp. 1–10. http://dx.doi.org/10.1155/2015/982350

Harsha, S.P., Sandeep, H., Prakash, R. (2003): "The effect of speed of balanced rotor on nonlinear vibrations associated with ball bearings", *International Journal of Mechanical Sciences*, Vol. 24, pp. 725–740. http://dx.doi.org/10.1016/S0020-7403(03)00064-X

Havel, J., Herout, A. (2010), "Yet faster ray-triangle intersection (using SSE4)", *IEEE transactions on visualization and computer graphics*, Vol. 16 No. 3, pp. 434–438. http://dx.doi.org/10.1109/TVCG.2009.73

Hoffenson, S., Dagman, R., Söderberg, R. (2013), "Tolerance Specification Optimization for Economic and Ecological Sustainability", In: Abramovici, M., Stark, R. (Ed.), *Smart Product Engineering*, Springer, Berling/Germany, Hamburg/Germany, pp. 865–874. http://dx.doi.org/10.1007/978-3-642-30817-8_85

ISO 492 (2014), *Rolling bearings – Radial bearings – Geometrical product specifications (GPS) and tolerance values*, International Organization for Standardization, Geneva/Switzerland.

ISO 5753-1 (2009): *Rolling Bearings – Internal clearance – Part 1: Radial internal clearance for radial bearings*, International Organization for Standardization, Geneva/Switzerland.

ISO/TR 1281-1 (2008), *Rolling bearings – Explanatory nodes on ISO 281 – Part 1: Basic dynamic load rating and basic rating life*, International Organization for Standardization, Geneva/Switzerland.

ISO/TS 16281 (2008), *Rolling bearings – Methods for calculating the modified reference rating life for universally loaded bearings*, International Organization for Standardization, Geneva/Switzerland.

Kockara, S., Halic, T., Bayrak C., Rowe, R. A. (2009), "Contact Detection Algorihms", *Journal of Computers*, Vol. 4 No. 10, pp. 1053–1063.

Mitrović, R. M., Atanasovska, I. D., Soldat N. D., Momčilović, D. B. (2015), "Effects of Operation Temperature on Thermal Expansion and Main Parameters of Radial Ball Bearings", *Thermal Science*, Vol. 19 No. 5, pp. 1835–1844. http://dx.doi.org/10.2298/TSCI141223091M

Oswald, F. B., Zaretsky, E. V., Poplawski, J. V. (2009), "Interference-Fit Life Factors for Roller Bearings", *Tribology Transactions*, Vol. 52 No. 4, pp. 415–426. http://dx.doi.org/10.1080/10402000802687890

Oswald, F. B., Zaretsky, E. V., Poplawski, J. V. (2012), "Effect of Internal Clearance on Load Distribution and Life of Radially Loaded Ball and Roller Bearings", *Tribology Transactions*, Vol. 55 No. 2, pp. 245–265. http://dx.doi.org/10.1080/10402004.2011.639050

Saltelli, A., Chan, K., Scott, E. M. (2000), *Sensitivity Analysis*, Wiley, Chichester/England.

Schaeffler Technologies GmbH & Co. KG (2012), *Rolling bearing – Ball bearings, Roller bearings, Needle roller bearings, Track rollers, Bearings for screw drives, Insert bearings/housing units, Bearing housings, Accessories*.

Schleich, B., Wartzack, S. (2013), "How to determine the influence of geometric deviations on elastic deformations and the structural performance?", *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, Vol. 227 No. 5, pp. 754–764. http://dx.doi.org/10.1177/0954405412468994

Schleich, B., Wartzack, S. (2014), "A discrete geometry approach for tolerance analysis of mechanism", *Mechanism and Machine Theory*, Vol. 77, pp. 148–163. http://dx.doi.org/10.1016/j.mechmachtheory.2014.02.013

Schmid, S. R., Hamrock B. J., Jacobson, B. O. (2014), *Fundamentals of Machine Elements*, CRC press, Hoboken/USA.

SKF Group (2013), *Rolling bearings*.

Thornton, A. C. (1999), "Variation Risk Management Using Modeling and Simulation", *Journal of Mechanical Design*, Vol. 121 No. 2, pp. 297-304. http://dx.doi.org/10.1115/1.2829457

Walter, M. S. J., Spruegel, T. C., Wartzack, S. (2015), "Least Cost Tolerance Allocation for Systems with Time-variant Deviations", *Procedia CIRP*, Vol. 27, pp. 1–9. http://dx.doi.org/10.1016/j.procir.2015.04.035

Wardle F. P. (1988), "Vibration Forces Produced by Waviness of the Rolling Surfaces of Thrust Loaded Ball Bearings Part 1: Theory", *Journal of Mechanical Engineering Science*, Vol. 202 No. 5, pp. 305–312.

Ye Z.-h., Wang L.-q. (2015), "Optimization analysis on assembly interference of cylindrical roller bearings", *Advances in Mechanical Engineering*, Vol. 7 No. 7, pp. 1–13. http://dx.doi.org/10.1177/1687814015593868

Zhang M., Anwer, N., Mathieu, L., Zhao, H. B. (2011), "A discrete geometry framework for geometrical product specifications", *21th CIRP Design Conference*, Daejoen/South Korea, March 2011, pp. 142–148.