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How to consider Over-constrained Assemblies with Gaps in Tolerance-Cost Optimization?

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Abstract

Exact Constraint Design is well established in design theory to ensure a robust and predictable system behavior. In practice, however, it is often inevitable to deviate from this basic concept so that parts are constrained several times for rigidity or resilience. In doing so, gaps between mating parts have to be added to ensure assemblability and functionality. Despite the essential impact of gaps on the total product functionality, occurring uncertainties in part positioning are mostly neglected in tolerance analysis. In the context of tolerance-cost optimization, they have not been studied in detail so far.

To overcome this drawback, this article addresses the research question how over-constrained systems with gaps can be considered in tolerance-cost optimization ensuring the identification of both reliable and cost-optimal tolerance values. Based on an initial discussion on the complexity of tolerance analysis of over-constrained systems with gaps, their impact on the results and the efficiency of tolerance-cost optimization are discussed. With the aim to ensure a sufficient model accuracy in time-consuming applications, a framework for tolerance-cost optimization considering assemblies with multiple gap configurations based on surrogate models is presented and applied to a case study of industrial complexity. In doing so, a novel framework for a proper and efficient modelling of systems with gaps in tolerance-cost optimization supporting both researchers and practitioners is presented.

Keywords: Tolerance-cost optimization; Least-cost tolerance allocation; Over-constrained assembly; Tolerance analysis

1. Introduction and motivation

The principle of Exact or Minimum Constraint Design is well known in literature and practice [1, 2]. By constraining each translational and rotational degree of freedom exactly once, the individual part positions and orientations are clearly defined with respect to each other in the assembly [1, 2].

Even though redundant constraints lead to less robust and variation-sensitive systems, this rule is often discarded in practice [2]. Parts are fixed several times to ensure rigidity and resilience leading to statical indeterminate, over-constrained assemblies [3–5]. With the aim to facilitate the mounting of over-constrained assemblies under variation, gaps are applied between the ambiguous mating assembly features [6, 7]. This in turn leads to uncertainties in part positioning since degrees of freedom are incorporated in the system [8].

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Despite their significant influence on product functionality [6], gaps in over-constrained systems are often neglected or oversimplified to avoid the application of challenging and computationally-intensive tolerance analysis methods [6, 9]. In addition, tolerance-cost optimization of assemblies with gaps have not thoroughly been studied in literature yet.

Motivated to close this research gap, this article pursues the question how over-constrained systems with gaps can be considered in tolerance-cost optimization achieving reliable results in reasonable computing times. Therefore, the state of the art and related work is presented in section 2. Subsequently, section 3 discusses the conflict of a proper and efficient consideration of over-constrained systems with gaps in tolerance analysis and tolerance-cost optimization. Based on the first insights of section 3.1, a novel tolerance-cost optimization framework for over-constrained systems with gaps is presented in section 3.2 and exemplarily applied to a case study in section 4. In doing so, this article intends to give recommendations to researchers and practitioners for an efficient and reliable tolerance-cost optimization of over-constrained assemblies with gaps. Finally, section 5 gives a brief conclusion and outlook.

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Nomenclature

a	Assemblability indicator
$C_{(\mathrm{sum})}$	(Cumulated) Manufacturing costs
COP	Coefficient of prognosis
f_{F}^{ζ}	Functionality function for configuration ζ
f_{C}	Tolerance-cost function
g	Generation
$h_{\mathrm{F/A}}$	Indicator function
j	Process number
n	Sample size for tolerance analysis
p	Individuum of population
RMS	Root-mean-square error
$\boldsymbol{t}, t_i$	Tolerance
$t_{ij}^{\mathrm{ub}}, t_{ij}^{\mathrm{lb}}$	Upper, lower tolerance boundary ij
USL, LSL	Upper, lower specification limit
x_{ij}	Machine selection parameter
X	Influence parameter
Y	Assembly response
$\hat{z}, z_{\max}$	Estimated, maximum non-conformance rate
η_p	Population size
η_g	Total number of generations
η_{D}	Sample size of DOE
η_{ζ}	Number of configurations
ζ	Configuration

2. State of the art and related work

On the one hand, gaps between mating parts are required to ensure the functionality and the assemblability of over-constrained assemblies under uncertainty [3, 6]. On the other hand, they lead to deviations in part positioning, which further influences the product functionality [6, 8]. For a realistic gap modelling in tolerance analysis, the space of all possible displacements in each single joint has to be determined [10]. Thus, it represents the set of all possible rigid body transformations [12], exemplarily expressed by small displacement torsors or vectors [13, 14], and is frequently addressed under the term gap hull [10, 11], clearance space [12] or clearance volume [14] in literature.

As a consequence, the estimation of the model behavior is a challenging task since the gaps of the multiple joints lead to a high number of physically feasible configurations [3, 6]. In contrast to iso-constrained assemblies, the assembly response has thus to be described by a function of the respective configuration of the assembly [3] . Therefore, it must be specified in which configuration the fulfillment of the predefined key characteristic (KC) is critical. [13]. However, the definition of an explicit assembly response function with respect to one or more configuration(s) is often not possible [16]. The quantifier notion is an useful mathematical formulation to express the functional and assembly requirements as a function of the configuration(s) [13, 15]. In doing so, the model behavior influenced by both dimensional and geometrical tolerances can implicitly be represented through a set of constraints and solved by optimization techniques [3, 16]. If form deviations should be taken into account, the application of sophisticated models, such as Skin Model Shapes in combination with different contact modelling approaches are needed for a realistic gap hull modelling [11, 17, 18]. These mathematical models serve as a basis for the statistical tolerance analysis and evaluation of the KC for the probabilistic assembly behavior. While first- and second order reliability methods can significantly reduce the computational effort [4, 6, 16], the usage of sampling techniques for tolerance analysis is often preferred due to their universal applicability [9, 15, 17]. Despite some open research questions, current approaches yet enable a tolerance evaluation with reliable results serving as a basis for subsequent manual tolerance re-allocations.

However, the lack of quantitative cost information and systematic procedures do not lead to a least-cost tolerance design and thus cannot meet the steadily growing requirements in industry [19, 20]. Tolerance-cost optimization overcomes this drawback by quantitatively incorporating the quality and the cost aspect and solving the tolerance-cost problem by optimization [20, 21]. Pushed by the emergence of powerful, stochastic optimization algorithms in combination with the high rise of computing powers, tolerance-cost optimization has continuously been enhanced so it can nowadays be applied to optimize complex products with interrelated KCs [22], and time-variant systems [21] with respect to cost and quality, but also to robust design [23, 24] considering both dimensional and geometrical tolerances in a statistical manner [25].

Nevertheless, tolerance-cost optimization currently reaches its limits when tolerance analysis gets too complex and computationally intensive. Consequently, multiple gap configurations are often neglected or merely oversimplified to ensure that an optimal tolerance allocation can be achieved in acceptable computing times. With exception of a few publications [9], the consideration of over-constrained assemblies with gaps has not thoroughly been discussed in context of tolerance-cost optimization so far.

3. Tolerance-cost optimization of over-constrained assemblies considering multiple assembly configurations

In the following, the complexity of tolerance analysis of over-constrained assemblies with gaps and its effects on tolerance-cost optimization are discussed. Subsequently, a comprehensive framework for tolerance-cost optimization of over-constrained assemblies with gaps including surrogate models is presented.

3.1. Problem Statement

Among numerous existing methods for statistical tolerance analysis, sampling techniques are mostly preferred, especially in industry, since they are problem-independently applicable and not limited to specific probability distributions [9, 20]. However, their main drawback can be seen in their high computational effort since a relatively large number of samples n are necessary to achieve reliable results, especially when analyzing low scrap rates in parts-per-million (ppm) [20]. Hence, the choice of the level of detail of the mathematical model is decisive. In general, there is a severe conflict between the choice of a suitable model including its resulting model uncertainty, which is defined as "the difference between the mathematical model and the actual behavior of the system" [26], and the required computational effort for their evaluation.

Focusing on over-constrained assemblies with gaps, this dilemma is further complicated for several reasons. By the incorporation of additional degrees of freedom caused by the several joint clearances, there is an infinite number of assembly configurations with respect to the resultant gap hulls. Thus, it is necessary to specify in which configurations the quality requirements have to be fulfilled for a given set of tolerances $\boldsymbol{t}$ assigned to the characteristics $\boldsymbol{X}$ (see Fig. 1). As a consequence, the assembly response has to be analyzed for all η_ζ explicitly defined configurations ζ using a suitable function f_{F}^{ζ} [15]. In a further step, the results must be interpreted with the aid of a suitable quality metric, such as the non-conformance rate $\hat{z}$ which is defined as the ratio of non-conform parts to the total batch size. Hence, the application of distribution-dependent estimation techniques to estimate the total non-conformance rate is severely complicated [20].

As a consequence, an empirical non-conformance estimation technique based on indicator functions h [9] is required to consider firstly if the assembly h_{A} can be assembled, i.e. if there exists a gap configuration where the assembly requirements (AR) are fulfilled since there are no part intersections, and secondly if the functional requirements h_{F}^{ζ} are fulfilled for the multiple ζ gap configurations (see Fig. 1):

$$\hat{z} = 1 - \frac{\sum_{i=1}^{n} \prod_{\zeta}^{\eta_\zeta} h_{\mathrm{F}}^{\zeta}(Y_i) \cdot h_{\mathrm{A}}(\boldsymbol{X_i})}{n}, \tag{1}$$

$$\text{with}: \; h_{\mathrm{F}}^{\zeta}(Y_i) = \begin{cases} 0 & \text{if} \quad Y_i < LSL_k \vee Y_i > USL_k. \\ 1 & \text{if} \quad LSL \leq Y_i \leq USL, \end{cases} \tag{2}$$

$$h_{\mathrm{A}}(\boldsymbol{X}_i) = \begin{cases} 1 & \text{if} \quad \text{AR is fulfilled} \\ 0 & \text{if} \quad \text{AR is not fulfilled} \end{cases} \tag{3}$$

Even though this approach is easy to implement, it requires a high number of samples to deliver reliable results [20].

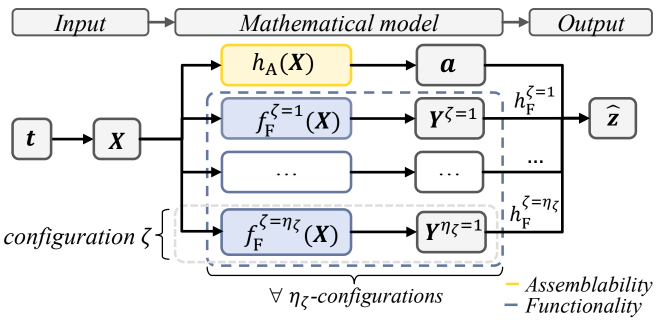

Fig. 1. Sampling-based tolerance analysis of over-constrained systems.

In most cases, however, it is either not possible or useful to explicitly define a fixed number of critical configurations. A suitable approach is needed to ensure functionality for all independent configurations caused by the probabilistic system behavior (see Fig. 1). In this regard, the formulation of a quantified constrained satisfaction problem and its solving using optimization algorithms has proven its suitability [13, 16, 18]. In doing so, the functionality is indirectly checked for all configurations since the worst-case configuration is identified to represent the total functionality [16]. However, it consequences high computing times since for each sample one optimization has to be performed. In combination with the inevitable need of high sample sizes, the calculation times of tolerance analysis can thus range between a few minutes up to days [16], depending on the chosen mathematical model, and is further intensified by the consideration of form deviations or elastic deformations [11, 18, 27].

When tolerance analysis has to be repeated due to revised tolerances or design parameters, the computational effort gets even more important. Consequently, this issue plays a decisive role in sampling-based tolerance-cost optimization which typically requires a huge number of re-allocations during the optimization procedure [20].

3.2. Sampling-based tolerance-cost optimization of over-constrained systems with gaps

In most cases, tolerance-cost optimization aims to identify the least-cost optimal tolerance values for a given tolerance specification [21, 22]. Thus, it corresponds to the identification of a set of tolerance values $\boldsymbol{t}$ that minimizes the resulting manufacturing costs C_{sum} while ensuring the fulfillment of predefined quality requirements typically measured by the aforementioned non-conformance rate $\hat{z}$ [20]:

$$\begin{aligned} &\text{Min} && C_{\mathrm{sum}}(\boldsymbol{t}) = \textstyle\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij} \cdot C_{ij}, \\ &\text{subject to:} && \hat{z}(\boldsymbol{t}) \leq z_{\max}, \\ & && x_{ij} \in \{0; 1\}, \end{aligned} \tag{4}$$

while the machine selection parameter x_{ij} chooses the cost-optimal process alternative with C_{ij} to realize the allocated tolerances. By applying sampling techniques in combination with an implicit formulation of the probabilistic system behavior of over-constrained assemblies, the optimization problem becomes noisy and complex to be solved. For this reason, stochastic, mostly population-based, derivative-free optimization algorithms are required to identify the optimum of the nonlinear objective constrained by nonlinear constraints. Even though the exact procedure depends on the chosen algorithm with its settings, the workflow for sampling-based tolerance-cost optimization of over-constrained assemblies, i.e. using sampling techniques for statistical tolerance analysis within the optimization framework [20], can generally be illustrated by Fig. 2.

Starting with an initial set of tolerances $\boldsymbol{t}_{\mathbf{init}}$, the optimizer iteratively generates a new set of tolerances $\boldsymbol{t}_p^g$ for each individual p. In a next step, the resultant costs $C_{p,\mathrm{sum}}^g$ are determined with the aid of a suitable tolerance-cost model f_{C} and

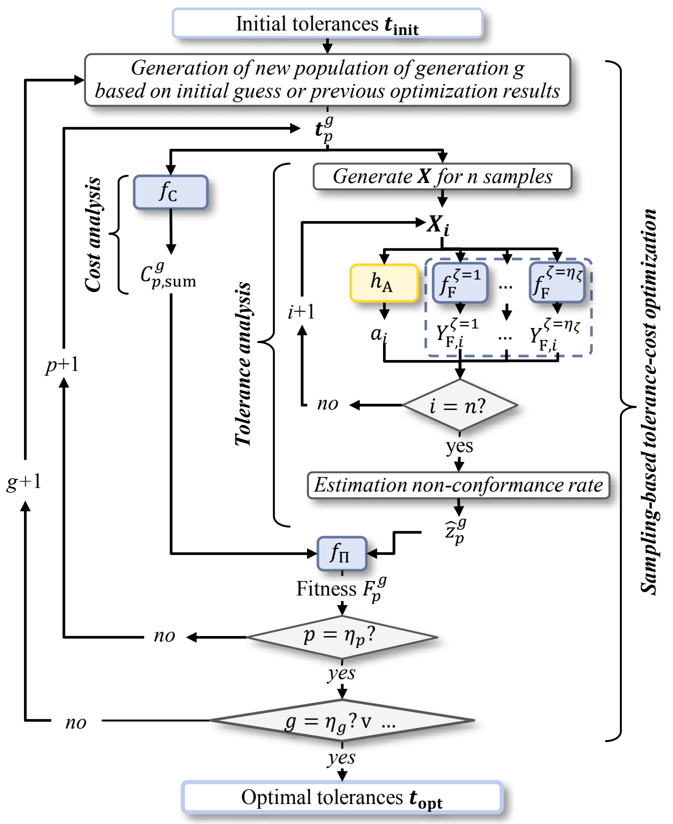

Fig. 2. General workflow of sampling-based tolerance-cost optimization of over-constrained assemblies with gaps.

the non-conformance rate $\hat{z}^g_p$ is estimated according to Eq. (1–3) for each individual t^g_p of the current generation g. The penalty function f_Π evaluates the fitness F^g_p of the current solutions. The best fitness values are used to create the new population. This procedure is repeated until a predefined termination criterion, e.g. a total number of generations η_g, is met and the optimal tolerances $\boldsymbol{t}_{\mathbf{opt}}$ are identified. For more details the reader is exemplarily referred to [9, 20].

As Fig. 2 clearly illustrates, the major drawback using population-based, stochastic optimization techniques for sampling-based tolerance-cost optimization can be seen in the high total number of function evaluations η_{total} for each individual p in each generation g for the sample size n in accordance to the algorithm-specific settings:

$$\eta_{\text{total}} = \eta_g \cdot \eta_p \cdot n. \quad (5)$$

As a consequence, the presented method requires a tremendous computing time which necessitates efficient countermeasures to ensure its applicability in practice. Focusing on the sample size n, its potential is limited since too low sample sizes lead to over- and underestimations and thus to either non-optimal or non-reliable tolerance values [20]. Furthermore, the choice of η_p and η_g (indirectly influenced by further algorithm-specific settings for termination criteria) is a demanding, problem-specific task. Too restrictive settings hinder a sufficient diversification, i.e. full exploration of the total design space, and intensification, i.e. the improving of a potentially good solution, and the global cost optimum cannot be reached [28]. As a consequence, the computation time for a single function evaluation in combination with its mathematical model has to be reduced to a minimum. However, an oversimplification of the problem is not purposeful since unrealistic simplifications cannot reflect its system behavior adequately (see section 3.1). Instead, surrogate models, also often called meta-models, provide a profitable alternative to replace time-consuming tasks within the optimization framework [30] and have already proven their general suitability in tolerancing, e.g. for the incorporation of thermal and mechanical effects in tolerance analysis [27, 29]. Hence, it is useful to substitute time-consuming mathematical functions g_A to evaluate the assemblability and the functionality f_F for all η_ζ configurations by approximative surrogate models (see Fig. 3).

Thereby, a suitable data basis in accordance to the given tolerance-cost optimization problem is needed for the subsequent meta-modelling process. In the first step, the upper and lower limits for each tolerance t_i have to be identified with respect to the given tolerance-cost model. Ensuring a full coverage of the design space for the optimization, the total definition range covering all machine alternatives j for tolerance t_i is defined by their minimum and maximum boundaries $t_i^{\text{lb}} = \min\left(t_{ij}^{\text{lb}}\right)$ or $t_i^{\text{ub}} = \max\left(t_{ij}^{\text{ub}}\right)$. Afterwards, a suitable design of experiment (DOE), e.g. a Latin-Hypercube-Sampling (LHS) with a suitable sample size η_D, is defined. For each sample, a tolerance analysis is carried out delivering information on both the assemblability $\boldsymbol{a}$ and the functionality $\boldsymbol{Y}$ for the given input characteristics $\boldsymbol{X}$. Since the assemblablity is expressed by a boolean bicriteria (see Eq. (3)), classification algorithms are suitable to create a surrogate model $\tilde{f}_A$ predicting whether an assembly with the characteristics $\boldsymbol{X}$ can be assembled or not. Applying regression techniques, surrogate models $\tilde{f}_F$ can be defined to predict the resulting value of the KC of a given $\boldsymbol{X}$. The quality of prognosis of the surrogate models has to be proven, e.g. by the root-mean-square error *RMS* or the coefficient of prognosis *COP* [31] for regression and by the accuracy for classification. In addition, it is useful to evaluate the resulting difference of the real and the predicted non-conformance rate $\Delta_{\hat{z}} = \left|\hat{z}(f_A, f_F) - \hat{z}(\tilde{f}_A, \tilde{f}_F)\right|$ since this criteria is essential for tolerance-cost optimization (see Eq. (4)). Finally, they can replace $\tilde{f}_F$ and $\tilde{f}_A$ in the optimization framework (see Fig. 2) and thus reduce the computational effort to enable sampling-based tolerance-cost optimization.

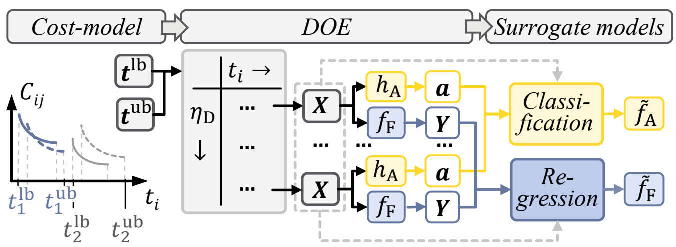

Fig. 3. Creating surrogate models to replace time-consuming mathematical models considering multiple gap configurations in tolerance-cost optimization.

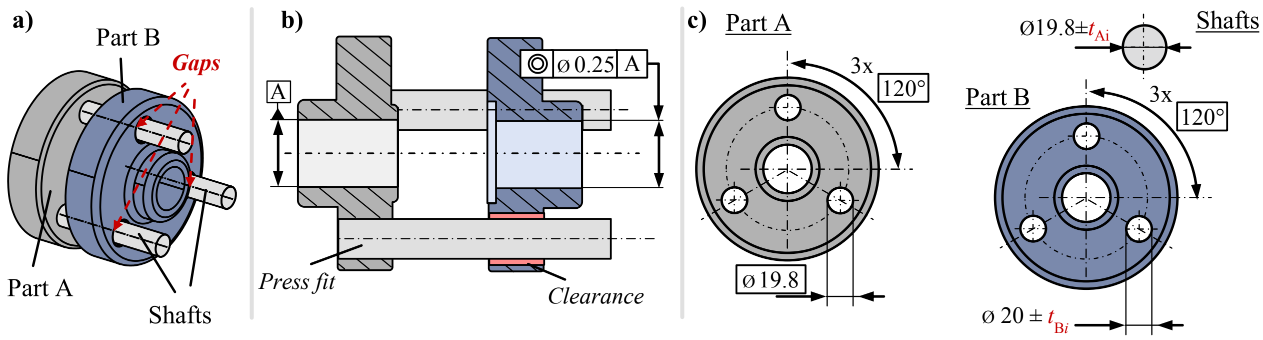

Fig. 4. Over-constrained assembly with gaps according to [15]: a) Overview, b) Key characteristic, c) Tolerance specification.

4. Application

In the following, the general suitability of the proposed approach is exemplarily studied for a simplified forging tool as a case study thoroughly presented in [15].

Presentation of the case study
The over-constrained assembly consists of two parts joined by three guide shafts with press fit in part A and floating contacts in part B (see Fig. 4a)). The coaxiality between the two center holes is considered as the KC with $USL = 0.25$ mm (see Fig. 4b)). A graph-based structure of the assembly helps to identify the topological loops serving as a basis to define the quantified constraint satisfaction problem by a set of non-linear equality and inequality conditions [13, 15]. This problem is solved by global numerical optimization technique while the gaps and deviations are represented by small displacement torsors in accordance to [15]. A LHS with a sample size of $n = 100\,000$ is used for tolerance analysis while the tolerances are considered as normally-distributed with $\sigma = t_{A/Bi}/6$ (see Fig. 4c)).

Creating the surrogate models
In general, it is useful to set the tolerances for the same features of the individual parts equal $t_{A/B1} = t_{A/B2} = t_{A/B3}$ to reduce the setup costs for the manufacturing. In the next step, the boundaries for the tolerance to be optimized have to be identified (see Fig. 3). In this case, the boundaries for the tolerances are set to $\boldsymbol{t}^{lb} = [0.01, 0.01]^T$ and $\boldsymbol{t}^{ub} = [0.40, 0.40]^T$. A LHS with $\eta_D = 100$ is used to create the data basis for the surrogate models $\tilde{f}_A$ and $\tilde{f}_C$ according to Fig. 3. In the next step, the data serves as a basis to study the quality of prognosis for different regression and classification algorithms.

Discussion of the results
For classification $\tilde{f}_A$, the application of the k-nearest-neighbour algorithm led to an accuracy of 98,3%. For the regression model $\tilde{f}_F$, support vector machines with a 80/20 split of training and testing data lead to a $RMS = 0.01$ mm, $COP = 95,4\%$. Table 1 further compares the resulting non-conformance rates and their difference $\Delta_{\hat{z}}$ for three selected samples (see Sec. 3.2). It can be seen, that the quality of prognosis varies over the design space of the optimization. In general it must be said, that the acceptance level has individually to be chosen with respect to the maximum non-conformance rate z_{max} according to the Six Sigma level, the chosen sample size n and the specification limits LSL, USL and differences in specification and functional limits have to be considered in compliance with ISO 14253-4. The usage of increasing sample sizes n over the optimization process can possibly further reduce the total computing time while improving the results (see Eq. 5)) [9].

Furthermore, the computing times could be reduced by the factor of approximately 100 illustrating that surrogate models are a profitable way to significantly reduce the total computing time for tolerance-cost optimization. Further studies including different use cases are needed to discuss strategies for an efficient setup for the data basis and studying the influence of the usage of different regression and classification algorithms in combination with the choice of the sample size η_D on the optimization results. In addition, other important aspects influencing the optimization results, e.g. measurement [32] and cost uncertainties [33], have to be considered to develop an holistic optimization approach.

Table 1. Comparison of results for selected tolerances.

$\boldsymbol{t}$	$\hat{z}(f_A, f_F)$	$\hat{z}(\tilde{f}_A, \tilde{f}_F)$	$\Delta_{\hat{z}}$
0.01, 0.01	0 ppm	0 ppm	0 ppm
0.20, 0.20	3170 ppm	2760 ppm	410 ppm
0.40, 0.40	76450 ppm	76720 ppm	270 ppm

5. Conclusion and outlook

Despite the increasing complexity of industrial applications, literature mostly focuses on tolerance-cost optimization of comparatively simple assemblies. In general, over-constrained systems with gaps are oversimplified to reduce the computational effort. However, unrealistic assumptions leading to high model uncertainties consequence an either non-optimal or unreliable tolerance allocation. With the aim to create a balance between model uncertainty and optimization efficiency, a novel approach for sampling-based tolerance-cost optimization based on surrogate models to represent the model behavior for an infinite number of possible assembly configurations was presented. Its exemplarily application proved its general suitability to reduce the computational effort.

Further research is essential to derive guidelines for the generation of a sufficiently large data basis for reliable surrogate models taking different classification and regression techniques into account. In combination with enhanced sampling methods, computing time of stochastic optimization can significantly be reduced and thus enables the usage of sampling methods for the optimization of assemblies with multiple gap configurations.

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